PowerApps – Optimal Power Flow Formulation

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1 OPF Problem Statement

Minimize $f(x, u)$ [cost function] (1)

Subject to constraints

$g(x, u) = 0$ [Load flow constraints] (2)

$umin \leq u \leq umax$ [Controller minimum and maximum limits] (3)
$xmin \leq x \leq xmax$ [Dependent variable minimum and maximum limits] (4)
$hmin \leq h \leq hmax$ [Functional Security Constraints] (5)

The cost function $f(x, u)$ may denote various objectives such as

1. Economic Dispatch [Generation cost]. In PowerApps, this problem is solved by continuously evaluating the operating cost, following generation allocation by a optimization technique in PowerApps. This document does not describe this algorithm and is confined to LP solution formulation for reactive power dispatch problem.
2. Transmission loss minimization [Mainly reactive power dispatch problem, implemented as minimization of slack generation when all other generation is held constant at specified values as needed in load flow solution. Active power injection control cannot be used with this objective, as they will minimize the slack generation, without any guarantee to loss minimization.]
3. Economic dispatch is solved first considering only active power generation control and the solution is used with reactive power dispatch using transmission loss minimization
4. System Security Improvement [Both active and reactive power dispatch may be involved]. These usually consist of limits on bus voltage magnitudes, limits on reactive power generations, limits on line loading.

Constraints defined by (2) are the power flow equations to be satisfied at any operating point. These (equation 2) simply denote that the load flow mismatch power must be 0 for specified operating point and for given values of x and u.

- The vector u is a set of control variables or independent variables.
- Vector x is a set of dependent variables. [load bus (PQ) voltage magnitudes and their phase angles, and phase angles of PV buses]
- Vector h is a set of security constraint variables.[bus voltage magnitude limits, Generation Q limits, line load limits]

1.1 Vector u

Vector u is a set of control variables which may comprise

1) Generator excitations$[E_g]$. Excitation is a control variable as it can be controlled by the AVR of the generator.
2) Constant MVAR type reactive power controls. Though most reactive power compensations are shunt capacitors or shunt reactors of constant impedance type, constant MVAR type compensation can be considered in view of the fact, that load powers are specified as constant powers for load flow problem. Thus constant MVAR type compensation will enable us to know the MVAR compensation needed for given load to improve the objective and meet constraints.
3) Transformer taps: The actual implementation uses inverse of the tap,$T = \frac{1}{\tau}$. This control is represented by the vectors,$[\tau]$, $[\tau_{max}]$, $[\tau_{min}]$, $[\tau_{step}]$. Algorithm will use these values in per unit, for calculation purpose.

4) Capacitive reactive power compensations of constant impedance type, like shunt capacitors. Constant impedance type compensations may be represented by vectors - $[b], [b_{max}], [b_{min}], [b_{step}]$. The limits are specified in MVAR and converted to susceptance within the PowerApps program on system base MVA.
5) Inductive reactive power compensations of constant impedance type like shunt reactors. Constant impedance type compensations may be represented by vectors - $[b], [b_{max}], [b_{min}], [b_{step}]$. Note the user may specify the compensation limits in MVAR and the same may be converted to susceptance format [b] on **system** MVA base.
6) Active power bus injections: These controllers may be denoted by the vectors$[p], [p_{max}], [p_{min}], [p_{step}]$, where all values are in per unit and on system base. These are currently restricted to economic dispatch function for generators only. In PowerApps the problem of economic dispatch using these controls are solved first, followed by reactive power dispatch using other specified controllers.
7) Series Compensation represented by vectors or variables as$[X], [X_{max}], [X_{min}], [X_{step}]$ on system base.
8) Phase shifter control. Similar to transformer, a phase shifter is represented by a complex turns' ratio. Where, $a = \frac{1}{\alpha}$. These variables may be represented by- $[\alpha], [\alpha_{max}], [\alpha_{min}], [\alpha_{step}]$.

1.1.1 Costs Associated with Vector [u] for Economic Dispatch

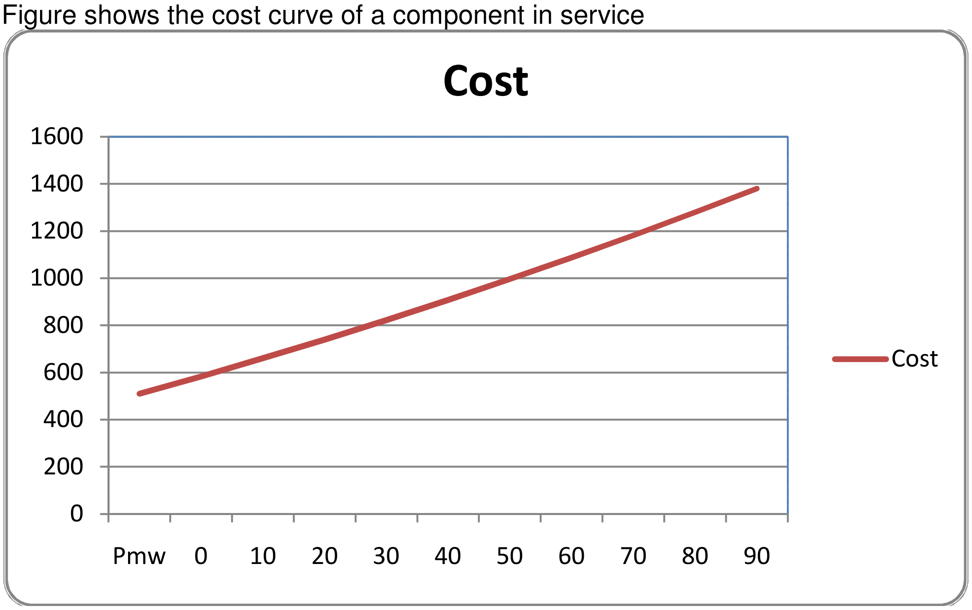
For Generator MW controllers specified in section 1.1 a common cost model based on MW power flow can be used. The cost model can be of the format

$$C = C_0 + C_1 P + C_2 P^2 \qquad (6)$$

Where, C_0, is a fixed cost in Currency/MW installed capacity, irrespective of the fact whether the particular unit/component is in service or not. This is a kind of facility charge to be levied to get returns on investment on facilities.

The costs, C_1 and C_2 are operating costs based on the operation of the unit/component and covers the cost of operation, maintenances and any other miscellaneous expenses

Figure shows the cost curve of a component in service



The coefficients of equation [6] can be obtained from the cost curves as shown in the figure using curve fitting techniques.

Since C0 is a fixed component, it remains constant and costs are minimized based on variable components C1 and C2 only.

1.1.2 Costs Associated with Transmission loss minimization

As explained previously slack generation is dependent variable in load flow jacobian as transmission loss is unknown quantity. Minimization transmission loss is equivalent to minimization of slack MW generation. Consequently we can express slack generation as function of other control variables and minimize the same for minimizing the transmission loss. This assumes that all other bus power specifications are specified and constant. This in turn implies MW power controls cannot bus used as controls when minimizing slack generation for purpose of loss minimization. If MW controls are used with slack bus power as minimization function, slack bus power will reduce without any guarantee of loss minimization.

1.1.3 Minimizing Transmission Cost/ or Cost of Specific Power Transaction

This cost model can follow the equation 6, where P refers to specific power transaction in a specified line or lines. P is to be expressed as function of other control variables as transmission line flows are dependent variable.

1.1.4 Minimizing the Bus Power Cost

Again this may follow the same cost model as equation [6]. Bus power need to be expressed as function of various controllers using sensitivity relations and the cost model has to be used.

1.1.5 ATC Calculation

With MW power controls specified along with any other controls indicated, we might try to maximize the line loading of specified tie-line, subject to security constraints. This provides an increase tie-line flow without violating any security constraint and indicates the Available transmission capacity of the line.

1.1.6 Tie Line Power Control

The security constraints of tie line flows are function of the various controls. The cost coefficients associated with a given tie line flow, indicates which controls are most efficient in controlling power flows in a given tie line. Consequently OPF can provide solutions to tie line power control.

1.1.7 Tracing of Tie Line Power Flows

The sensitivities of tie line power flows with respect to generation MW controls indicate the participation of the various generation companies and load centers in the power flow of a particular tie-line flow. This information is likely to be useful while trying to minimize the cost of the transmission or determining the payments to be made to the generation companies from end consumers.

1.2 Vector x

Vector x denotes the dependent variables state variables, bus voltage magnitude and phase angles of all PQ buses in the system. Further bus voltage angles of PV buses are also part of the vector x. The slack bus voltage angle is a fixed parameter.

Most other dependent variables such as reactive power generation, slack generation, line flows etc. are expressed in terms of these dependent as well as the control variables.

From the consumer point of view, phase angle does not have much significance and these do not appear directly in any constraints or objective equation in direct manner. However, from the quality of power supply point of view, bus voltage magnitudes needs to be maintained within acceptable tolerances. Thus we need to specify the limits on bus voltage in per unit. Typically these limits are 1.05 per unit for upper limit and 0.95 per unit for lower limit. These voltage magnitude limits of all PQ buses may be represented by vectors $[v], [v_{max}], [v_{min}]$

1.3 Vector h

These are security limits and comprises of

- Reactive power limits of the Generators. These limits may be handled with vectors$[qg], [qg_{max}], [qg_{min}]$.
- MW flows through lines at given operating power factor. MVA flow sensitivities needs to be worked out if we have to handle MVA flows as the basis for the security limit checks. These limits may be handled with vectors $[h], [h_{max}], [h_{min}]$.
- MW flows through transformers at given operating power factor. MVA flow sensitivities needs to be worked out if we have to handle MVA flows as the basis for the security limit checks. These limits may be handled with vectors $[h], [h_{max}], [h_{min}]$
- Specified MW flow through phase shifter. This may not be necessarily security limit, but an equality constraint for scheduled power flow in a given line. By specifying the limits equal to the scheduled quantity, we may handle this similar to security limits. These limits may be handled with vectors $[h], [h_{max}], [h_{min}]$

Note: Scheduled power exchange over line can be specified as equality constraint with maximum and minimum limits specified as same value, making it an equality constraint.

2 Reduced Model Formulation

The equation

$$g(x, u) = 0 \quad (2),$$

Can be linearized around the power flow solution [Where the mismatch is 0 or minimum] to get the following equations

$$\left[\frac{\partial g}{\partial x}\right] \Delta x + \left[\frac{\partial g}{\partial u}\right] \Delta u = 0 \quad (7)$$

or

$$\Delta x = -\left[\frac{\partial g}{\partial x}\right]^{-1} \left[\frac{\partial g}{\partial u}\right] \Delta u \quad (8a)$$

or

$$\Delta x = [S_x] \Delta u \quad (8b)$$

Where,

$$[S_x] = -\left[\frac{\partial g}{\partial x}\right]^{-1} \left[\frac{\partial g}{\partial u}\right] \quad (8c)$$

Note the inverse $\left[\frac{\partial g}{\partial x}\right]^{-1}$ is simply the inverse of the power flow jacobian at the operating point, under consideration.

The x in equation 2, 7, 8 denotes following dependent variables.

- Bus voltage angle of all buses [except the slack bus voltage angle], which is invariant and a fixed specified parameter during load flow solution.
- Bus voltage magnitude of all PQ buses [except for slack and PV Generator bus, which are control variables].

Note that even when load flow converts some PV buses to PQ buses to satisfy the reactive power limit, the generator buses are always treated as PV bus for purpose of sensitivity calculations as per equation (8).

If $Nbus$ is the number of buses and $Ngen$ is the number of generators, then, x will have

- ⇨ $Nbus - 1$, entries for bus voltage angle.
- ⇨ $Nbus - Ngen$, number of entries for the bus voltage magnitudes.

Again we notice that g is made of p, q and u is made of several type of controllers. Consequently we may write equation (8a) as follows [written in the form of a single bus, each entries however, denotes a vector, or matrix as applicable.]

$$\begin{bmatrix} \Delta\delta \\ \frac{\Delta E}{|E|} \end{bmatrix} = -\begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |E|}|E| \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |E|}|E| \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial P}{\partial |E_g|}\Delta|E_g| & \frac{\partial P}{\partial b} & \frac{\partial P}{\partial \tau} & \frac{\partial P}{\partial q} & \frac{\partial P}{\partial X} & \frac{\partial P}{\partial \alpha} \\ \frac{\partial Q}{\partial |E_g|}\Delta|E_g| & \frac{\partial Q}{\partial b} & \frac{\partial Q}{\partial \tau} & \frac{\partial Q}{\partial q} & \frac{\partial Q}{\partial X} & \frac{\partial Q}{\partial \alpha} \end{bmatrix} \begin{bmatrix} \frac{\Delta|E_g|}{|E_g|} \\ \Delta b \\ \Delta\tau \\ \Delta q \\ \Delta X \\ \Delta\alpha \end{bmatrix} \quad (8d)$$

Each entry in (8d) is either a matrix or vector. We need to compute all the values of (8d) in an efficient way. In actual implementation, equation (8d) is formulated as pairs of $[\Delta\delta, \frac{\Delta E}{|E|}]$ for each bus [P,Q], i.e these variables appear alternatively in the matrix, rather than as separate vector's or matrices. The exact structure of [8d] for a 2 bus system, connected between bus "i" and bus "j" and having two controllers' u1 and u2 will be as follows

$$\begin{bmatrix} \Delta\delta_i \\ \frac{\Delta E_i}{|E_i|} \\ \Delta\delta_j \\ \frac{\Delta E_j}{|E_j|} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_i}{\partial \delta_i} & \frac{\partial P_i}{\partial |E_i|}|E_i| & \frac{\partial P_i}{\partial \delta_j} & \frac{\partial P_i}{\partial |E_j|}|E_j| \\ \frac{\partial Q_i}{\partial \delta_i} & \frac{\partial Q_i}{\partial |E_i|}|E_i| & \frac{\partial Q_i}{\partial \delta_j} & \frac{\partial Q_i}{\partial |E_j|}|E_j| \\ \frac{\partial P_j}{\partial \delta_i} & \frac{\partial P_j}{\partial |E_i|}|E_i| & \frac{\partial P_j}{\partial \delta_j} & \frac{\partial P_j}{\partial |E_j|}|E_j| \\ \frac{\partial Q_j}{\partial \delta_i} & \frac{\partial Q_j}{\partial |E_i|}|E_i| & \frac{\partial Q_j}{\partial \delta_j} & \frac{\partial Q_j}{\partial |E_j|}|E_j| \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial P_i}{\partial u_1} & \frac{\partial P_i}{\partial u_2} \\ \frac{\partial Q_i}{\partial u_1} & \frac{\partial Q_i}{\partial u_2} \\ \frac{\partial P_j}{\partial u_1} & \frac{\partial P_j}{\partial u_2} \\ \frac{\partial Q_j}{\partial u_1} & \frac{\partial Q_j}{\partial u_2} \end{bmatrix} \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix} \quad (8e)$$

Note that for any bus voltage angle, or voltage magnitude, we determine sensitivity with respect to any given controller from (8e). [Of course, we will not have entries corresponding to slack bus voltage angle and voltage magnitudes corresponding to each generator bus and slack bus, as these do not exist in the load flow jacobian]

From the equation (8c) and (8e) it is seen that $[S_x]$ has a dimension of 2Nbus*Nu, where Nbus is number of buses and u is number of controllers. Thus to get (i,j) element of the $[S_x]$ we only need to multiply i^{th} row of $-\left[\frac{\partial g}{\partial x}\right]^{-1}$ with j^{th} column of $\left[\frac{\partial g}{\partial u}\right]$.The calculation of $[S_x]$, is therefore done without using full matrix technique. Instead using sparse matrix technique, the required row of $-\left[\frac{\partial g}{\partial x}\right]^{-1}$, is generated and is multiplied with required column of $\left[\frac{\partial g}{\partial u}\right]$. Further the matrix $\left[\frac{\partial g}{\partial u}\right]$ is also sparse and only non-zero elements of this matrix need to be stored in a compact format.

If Nbus is the total number of buses, Nu is the total number of controllers, the first matrix on the RHS of (8d) is 2Nbus*2Nbus dimension. The second matrix on RHS of (8d) is [2Nbus*Nu] dimension. The resulting product is 2Nbus*Nu dimension. We only need to use this resulting matrix dimension for storing required information with size [2Nbus*Nu]. The same matrix may be later used for LP Tableau after all sensitivities are computed. Organizing this computer memory storage efficiently is the important requirement in the OPF software development.

2.1 Transmission Loss Minimization by Minimizing the Slack Generation

If load flow power specification for generators is assumed to be obtained from an economic dispatch and specified MW generations for PV buses are therefore cannot be changed as control variables, Minimization of the transmission losses becomes the same problem as minimization of the slack generation. It may be noted that slack generation cannot be specified before the load flow solution as the losses are unknown. Lesser is the loss, lesser will be slack generation. In other words, any objective that minimizes the slack generation is equivalent to minimizing the transmission losses. Since real power specifications are fixed in load flow, minimization of the slack generation or minimization of the transmission losses is considered as a reactive power dispatch problem. The losses are minimized by minimizing the reactive power flow in the network.

The slack generation $P_{sl} = f(x, u)$ is function of dependable variables x and control variables u
The changes in slack generation due to changes in x and u is therefore given by

$$\Delta P_{sl} = \left[\frac{\partial P_{sl}}{\partial x}\right]^T \Delta x + \left[\frac{\partial P_{sl}}{\partial u}\right]^T \Delta u \qquad (9a)$$

$$\Delta P_{sl} = \left[\frac{\partial P_{sl}}{\partial x}\right]^T [S_x]\Delta u + \left[\frac{\partial P_{sl}}{\partial u}\right]^T \Delta u \qquad (9b)$$

$$\Delta P_{sl} = \left[\left[\frac{\partial P_{sl}}{\partial x}\right]^T [S_x] + \left[\frac{\partial P_{sl}}{\partial u}\right]^T\right] \Delta u \qquad (9c)$$

Thus we minimize the slack generation, ΔP_{sl} , which is equivalent to loss minimization, under the assumption that other generations are fixed and invariable. The coefficients of Δu, in (9c) are the required cost coefficient in the objective function. The cost coefficient having highest magnitude evidently denotes the control variable of highest influence on the objective function.

Note again, that in this problem, all MW generations are fixed from economic dispatch and cannot be changed. Thus this problem is equivalent to optimal reactive power dispatch, with all MW generations held constant.

Note that the vector u, in this problem does not have active power injection as a control variable.

Typically among the vector u, only the slack generation excitation control $\Delta|E_g|$ influences the slack generation,P_{sl}. No other controllers are likely to be connected directly to the slack bus.

Note we **should not** specify generator transformer tap controller and Generator excitation controller together as they have the same similar role to play. Both these controllers influence the reactive power output similarly. Consequently tap controllers are specified for ICT's or for transformers connected between two PQ buses.

The term $\left[\frac{\partial P_{sl}}{\partial u}\right]^T \Delta u$ in equation (9b) therefore usually represent only the slack bus $\Delta|E_g|$. The effect of the other controllers on slack generation is accounted by the term $\left[\frac{\partial P_{sl}}{\partial x}\right]^T [S_x]\Delta u$. However, care must be taken to account for the direct influence of any other controller on slack generation.

No MW Generation Controller should be used with this objective, as the assumption is all MW generation are fixed and minimization of slack generation is the minimization of system losses.

2.2 Economic Dispatch with Network Constraints

This section discusses generic concepts, which is not completely implemented in PowerApps.

If there are no major security constraint violations in the solution of section 2.1, we may say that we have the optimal solution. But security constraints such as the following may exist

List of unacceptable operating conditions

- **Unacceptable overload conditions**
- **Too much deviation in scheduled tie line flows**
- **Unacceptable bus voltages**
- **Reactive power generation limit violations of the generators**

[Note ABT constraints fall under the above. ABT is also related to frequency. Frequency is not directly handled in OPF which is a static analytical model, where frequency cannot be modeled. However, frequency is related to power flows and it is usually possible to predict the loading condition and resulting frequency, based on system operating condition observation in energy control centers. These observations are used in imposing the line load limits and operating limits to control frequencies]

It may now be necessary to introduce additional controllers like MW generations, which are ignored in the section 2.1

In addition the transmission cost is omitted in the section 2.1 and the economic dispatch was considered purely the cost of generation only. However, in the present day – deregulated market, the cost of transmission, distribution and generation are all different. This cost is now available for all components as per section 1.1.1.

Thus our new cost function may be defined us

$$C = \sum_{i=1}^{number\ of\ components} C_{0i} + C_{1i}P_i + C_{2i}P_i^2 \qquad [9d]$$

The above equations have both control variables u[Bus power injections from generations Pi] and dependent variables [dependent Pi of components whose cost also has to be minimized, say, transmission cost – May comprise transmission line flows, Load bus powers].
Except for the slack generation, remaining generations are modeled as control variables in load flow jacobian. The dependent variables of slack generation [Refer equation (9c)] and line flows [Refer equation 11(b) and 11(d)] must be expressed in terms of other controllers and used in equation [9d].

2.2.1 Minimization of Generation Cost or Cost of Production

Let P_i be the i^{th}, generation with cost model $C_i = C_{0i} + C_{1i}P_i + C_{2i}P_i^2$, Then, the variation of the cost of i^{th} generation with respect to the MW generation is

$$\frac{\partial C_i}{\partial P_i} = C_{1i} + 2C_{2i}P_i$$

Therefore the change in the cost of generation of i^{th} generator is given by

$$\Delta C_i = \frac{\partial C_i}{\partial P_i}\Delta P_i = (C_{1i} + 2C_{2i}P_i)\Delta P_i \qquad [9e]$$

To minimize the total cost of generation in the system we need to minimize the sum of all expressions similar to [9e], thus our objective will be

Minimize

$$\Delta C = \sum_{i=1}^{i=n} \Delta C_i = \sum_{i=0}^{i=1} \frac{\partial C_i}{\partial P_i}\Delta P_i = \sum_{i=1}^{i=n} (C_{1i} + 2C_{2i}P_i)\Delta P_i \qquad [9f]$$

Where n is the number of generation control whose cost is to be minimized. The slack generation is expressed in terms of other controllers and used in [9f].

2.2.2 Handling Slack Generation Cost

Without doubt, the cost of the slack generation must be included in [9f]. However the slack generation is dependent on other controllers and its relation with other controllers is given by the expression

$$\Delta P_{sl} = \left[\left[\frac{\partial P_{sl}}{\partial x}\right]^T [S_x] + \left[\frac{\partial P_{sl}}{\partial u}\right]^T\right]\Delta u \qquad (9c)$$

Consequently the entire expression of (9c) must be multiplies with $(C_{1sl} + 2C_{2sl}P_i)$ and later added to equation [9f] to obtain the objective function.

2.3 Security Constraints

The security constraints on the optimal power flow is defined by the constraints

$$hmin \leq h \leq hmax \qquad (5)$$

Where

$$h = h(x, u) \qquad (10)$$

Consequently the linearized relation of the h is given by

$$\Delta h = \left[\frac{\partial h}{\partial x}\right]\Delta x + \left[\frac{\partial h}{\partial u}\right]\Delta u \qquad (11a)$$

Using equation 8b, 11 can be written as

$$\Delta h = \left[\left[\frac{\partial h}{\partial x}\right][S_x] + \left[\frac{\partial h}{\partial u}\right]\right]\Delta u \qquad (11b)$$

Elements of h may comprise reactive power limits of generation, line loading limits [either MW or MVA] as function of control variables.

2.3.1 Handling MVA limits

The power flow jacobian handles active and reactive power mismatches and provide active and reactive power sensitivities. Similarly sensitivity of active and reactive power flows can also be obtained from the power flow expressions. To handle MVA limits we need to use the following relations

$$mva = (p^2 + q^2)^{1/2} \tag{11c}$$

$$\Delta mva = \frac{\partial mva}{\partial p}\Delta p + \frac{\partial mva}{\partial q}\Delta q \tag{11d}$$

$$\frac{\partial mva}{\partial p} = \frac{1}{2}(p^2 + q^2)^{-1/2}.2p \tag{11e}$$

Since,

$$mva = (p^2 + q^2)^{-1/2} \tag{11f}$$

We get

$$\frac{\partial mva}{\partial p} = mva.p \tag{11g}$$

$$\frac{\partial mva}{\partial q} = mva.q \tag{11h}$$

Thus if we now mva flow, active power flow p and reactive power flow q, and sensitivity of active power flow and reactive power flow with respect to [x] and [u], we can formulate mva flow limit constraints in the OPF problem.

2.4 Sensitivities

2.4.1 Sensitivities of Bus Powers with respect to bus voltage magnitudes and angles

The sensitivities of the bus power injections with respect to dependent bus voltage magnitude and angles $\{|E|, \delta\}$ and independent bus voltage magnitudes [generator excitation specifications$\{|E_g|\}$] are obtained by using partial derivatives as used in formulation of power flow jacobian [1]. The following notations [1] are used in partial derivatives. The sensitivity with respect to the slack bus voltage angle [reference bus] is ignored as it is invariant and a reference angle.

$$Y_{km} = (G_{km} + jB_{km}); \tag{12a}$$

$$E_m = (e_m + jf_m) \tag{12b}$$

$$I_m = (a_m + jb_m) \tag{12c}$$

$$[Y][E] = [I] \tag{12d}$$

The partial derivatives when$k \neq m$, are given by

$$\frac{\partial P_k}{\partial \delta_m} = \frac{\partial Q_k}{\partial |E_m|}|E_m| = a_m f_k - b_m e_k \tag{13a}$$

$$-\frac{\partial Q_k}{\partial \delta_m} = \frac{\partial P_k}{\partial |E_m|}|E_m| = a_m e_k + b_m f_k \tag{13b}$$

The partial derivatives when$k = m$, are given by

$$\frac{\partial P_k}{\partial \delta_k} = -Q_k - B_{kk}|E_k|^2 \tag{13c}$$

$$\frac{\partial Q_k}{\partial \delta_k} = P_k - G_{kk}|E_k|^2 \tag{13d}$$

$$\frac{\partial P_k}{\partial |E_k|}|E_k| = P_k + G_{kk}|E_k|^2 \tag{13e}$$

$$\frac{\partial Q_k}{\partial |E_k|}|E_k| = Q_k - B_{kk}|E_k|^2 \tag{13f}$$

Equations 13a to 13f provides the required sensitivity of the bus power injections with respect to the bus voltage magnitude and their phase angles. Note that these sensitivities are same as load flow jacobian elements.

2.4.2 Sensitivities of Bus Powers with respect to shunt reactive power compensation

Let,B_{sh}, be the per unit susceptance of the shunt reactive power compensation provided at a bus and V_{sh} be the bus voltage magnitude in per unit. The reactive power injected **in to** the bus by the shunt element is given by

$$Q_{sh} = -|E_{sh}|^2 B_{sh} \tag{14a}$$

The sensitivity of the reactive power absorption as a function of shunt susceptance is given by

$$\frac{\partial Q_{sh}}{\partial B_{sh}} = -|E_{sh}|^2 \tag{14b}$$

For constant power compensation the right hand side of equation (14b) becomes -1.0

Equations (14a) and (14b) influences only the reactive power bus injections at the bus to which the compensating equipment is connected. The equations (14a) and (14b) correspond to $\left[\frac{\partial g}{\partial u}\right]\Delta u$ portion of the equation (7).

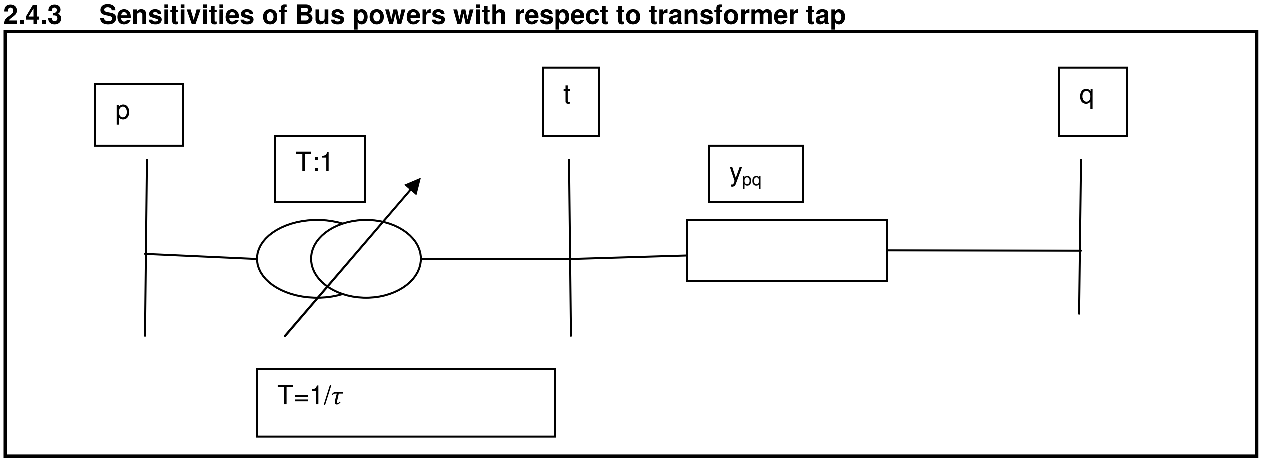
- **Note: Q_{sh} in 14(a) denotes power flowing out of the bus, which is positive for inductor [with –ve B_{sh}] and Q_{sh} is negative for capacitors [with +ve B_{sh}], with these notations the derivative $\frac{\partial Q_{sh}}{\partial B_{sh}}$, will have similar conventions like line flows.**

2.4.2.1 Comparison of equation (14b) with equations (15k)

In the next section sensitivity of the reactive power flow in transformer is given. The transformer is connected between buses p and q. If the transformer tap is 1 and bus q is grounded [voltage magnitude and angle 0], the equations (15k) represents equation for a shunt reactor as follows, For a pure reactor sine component will be -90^0 and equal to -1.0 and y_{pq} denotes susceptance B_{pq}. Thus (15k) will be negative to that of (14a) for pure reactor.

$$q_{pq} = Imag\{E_t i_{tq}^*\} = -\left|E_p^2 y_{pq}\right| \sin(\theta_{pq}) \quad = \left|E_p\right|^2 B_{pq} \tag{15k}$$

2.4.3 Sensitivities of Bus powers with respect to transformer tap



Let p and q be the transformer terminal buses with the off nominal turns ratio T:1, with the relation $T = \frac{1}{\tau}$. The series admittance of the transformer is y_{pq}. 't' is the fictitious node representing the terminal of ideal transformer T: 1 turns ratio.

The following relations applies

$$\frac{|E_p|}{|E_t|} = T = \frac{1}{\tau} \qquad (15a)$$

Equation (15a) is a pure "in-phase" transformation. i.e. there is no phase angle difference between the voltages E_p and E_t

$$y_{pq} = (g_{pq} + jb_{pq}) = |y_{pq}|e^{j\theta_{pq}} \qquad (15b)$$

$$E_p = |E_p|e^{j\delta_p} \qquad (15c)$$
$$E_q = |E_q|e^{j\delta_q} \qquad (15d)$$
$$i_{tq} = \left(|E_t|e^{j\delta_t} - |E_q|e^{j\delta_q}\right)|y_{pq}|e^{j\theta_{pq}} \qquad (15e)$$
$$i_{tq} = |E_t|e^{j\delta_t}|y_{pq}|e^{j\theta_{pq}} - |E_q|e^{j\delta_q}|y_{pq}|e^{j\theta_{pq}} \qquad (15f)$$
$$i_{tq} = |E_t y_{pq}|e^{j(\delta_t+\theta_{pq})} - |E_q y_{pq}|e^{j(\delta_q+\theta_{pq})} \qquad (15g)$$
$$i_{qt} = |E_q y_{pq}|e^{j(\delta_q+\theta_{pq})} - |E_t y_{pq}|e^{j(\delta_t+\theta_{pq})} \qquad (15h)$$
$$s_{tq} = E_t i_{tq}^* = |E_t^2 y_{pq}|e^{j(-\theta_{pq})} - |E_t E_q y_{pq}|e^{j(\delta_t-\delta_q-\theta_{pq})} \qquad (15g)$$
$$s_{qt} = E_q i_{qt}^* = |E_q^2 y_{pq}|e^{j(-\theta_{pq})} - |E_q E_t y_{pq}|e^{j(\delta_q-\delta_t-\theta_{pq})} \qquad (15h)$$

Now we use the relation from (15a) in (15g) and (15h). i.e. $|E_t| = \tau|E_p|$ and $\delta_t = \delta_p$. Further by changing the subscript, 't' to 'p', appropriately and separating real and imaginary parts we get the following

$$p_{pq} = Real\{E_t i_{tq}^*\} = |\tau^2 E_p^2 y_{pq}|\cos(\theta_{pq}) - |\tau E_p E_q y_{pq}|cos(\delta_p - \delta_q - \theta_{pq}) \qquad (15i)$$
$$p_{qp} = Real\{E_q i_{qt}^*\} = |E_q^2 y_{pq}|cos(\theta_{pq}) - |\tau E_p E_q y_{pq}|cos(\delta_q - \delta_p - \theta_{pq}) \qquad (15j)$$
$$q_{pq} = Imag\{E_t i_{tq}^*\} = -|\tau^2 E_p^2 y_{pq}|\sin(\theta_{pq}) - |\tau E_p E_q y_{pq}|sin(\delta_p - \delta_q - \theta_{pq}) \qquad (15k)$$
$$q_{qp} = Imag\{E_q i_{qt}^*\} = -|E_q^2 y_{pq}|\sin(\theta_{pq}) - |\tau E_p E_q y_{pq}|sin(\delta_q - \delta_p - \theta_{pq}) \qquad (15l)$$

[A special case of 15k for shunt compensation is found by setting Eq=0, $\tau = 1$, which gives equation (14a)]

The partial derivative equations of the flows with respect to the variable τ is given by the following

$$\frac{\partial p_{pq}}{\partial \tau} = 2|\tau E_p^2 y_{pq}|\cos(\theta_{pq}) - |E_p E_q y_{pq}|cos(\delta_p - \delta_q - \theta_{pq}) \qquad (15m)$$
$$\frac{\partial p_{qp}}{\partial \tau} = -|E_p E_q y_{pq}|cos(\delta_q - \delta_p - \theta_{pq}) \qquad (15n)$$
$$\frac{\partial q_{pq}}{\partial \tau} = -2|\tau E_p^2 y_{pq}|\sin(\theta_{pq}) - |E_p E_q y_{pq}|sin(\delta_p - \delta_q - \theta_{pq}) \qquad (15o)$$
$$\frac{\partial q_{qp}}{\partial \tau} = -|E_p E_q y_{pq}|sin(\delta_q - \delta_p - \theta_{pq}) \qquad (15p)$$

From equation (15m, 15n, 15o, 15p), it is seen that bus powers P and Q of "from (p)" and "to (q)" buses are affected. This information from equations (15m to 15p) is part of equation$\left[\frac{\partial g}{\partial u}\right]\Delta u$.

2.4.4 Sensitivities with respect to controlled series compensation

If B is the susceptance in per unit [negative for inductance and positive for capacitance], connected between the two buses p and q, the following power flow equations apply.

[Note series compensation is a special case of equation 15 with the following changes
$\tau = 1\ and\ E_t = E_p$; $y_{pq} = (0 + jb_{pq}) = |y_{pq}|e^{j90^0}; g_{pq} = 0$, Consequently equations of 15 becomes as follows]

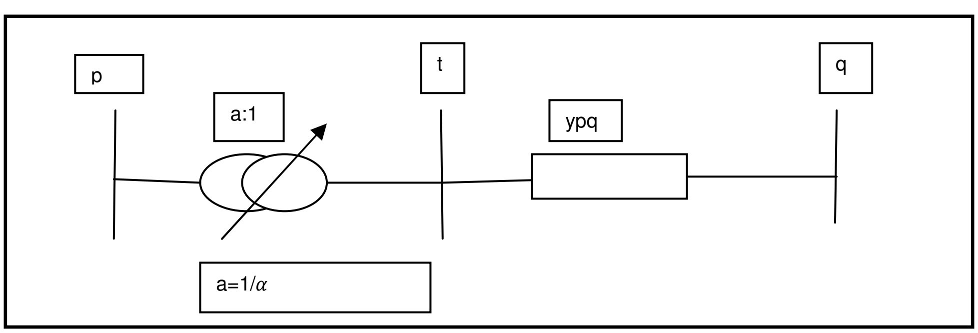
$$i_{pq} = \left(|E_p|e^{j\delta_p} - |E_q|e^{j\delta_q}\right)jb_{pq} \quad (16a)$$
$$i_{qp} = \left(|E_q|e^{j\delta_q} - |E_p|e^{j\delta_p}\right)jb_{pq} \quad (16b)$$
$$s_{pq} = E_p i_{pq}^* = -jb_{pq}|E_p^2| + jb_{pq}|E_pE_q|e^{j(\delta_p-\delta_q)} \quad (16c)$$
$$s_{qp} = E_q i_{qp}^* = -jb_{pq}|E_q^2| + jb_{pq}|E_pE_q|e^{j(\delta_q-\delta_p)} \quad (16d)$$
$$p_{pq} = Real\{E_p i_{pq}^*\} = -b_{pq}|E_pE_q|sin(\delta_p - \delta_q) \quad (16e)$$
$$p_{qp} = Real\{E_q i_{qp}^*\} = -b_{pq}|E_pE_q|sin(\delta_q - \delta_p) \quad (16f)$$
$$q_{pq} = Imag\{E_p i_{pq}^*\} = -b_{pq}|E_p^2| + b_{pq}|E_pE_q|cos(\delta_p - \delta_q) \quad (16g)$$
$$q_{qp} = Imag\{E_q i_{qt}^*\} = -b_{pq}|E_q^2| + b_{pq}|E_pE_q|cos(\delta_q - \delta_p) \quad (16h)$$

The partial derivatives of the power flow equations with respect to b_{pq} are as follows

$$\boldsymbol{\frac{\partial p_{pq}}{\partial b_{pq}} = -|E_pE_q|sin(\delta_p - \delta_q)} \quad \textbf{(16i)}$$
$$\boldsymbol{\frac{\partial p_{qp}}{\partial b_{pq}} = -|E_pE_q|sin(\delta_q - \delta_p)} \quad \textbf{(16j)}$$
$$\boldsymbol{\frac{\partial q_{pq}}{\partial b_{pq}} = -|E_p^2| + |E_pE_q|cos(\delta_p - \delta_q)} \quad \textbf{(16k)}$$
$$\boldsymbol{\frac{\partial q_{qp}}{\partial b_{pq}} = -|E_q^2| + |E_pE_q|cos(\delta_q - \delta_p)} \quad \textbf{(16l)}$$

2.4.5 Sensitivities with respect to phase shifters

Phase shifters are primarily used to control active power flow. The phase shifting transformer is represented by its admittance y_{pq} in series with the ideal auto transformer having a **complex** turn's ratio $a{:}1$



The following equation gives the mathematical model of the phase shifter

$$\begin{bmatrix} i_{pq} \\ i_{qp} \end{bmatrix} = \begin{bmatrix} \frac{y_{pq}}{aa^*} & -\frac{y_{pq}}{a^*} \\ -\frac{y_{pq}}{a} & y_{pq} \end{bmatrix} \begin{bmatrix} E_p \\ E_q \end{bmatrix} \quad (17a)$$

Let, $= \frac{1}{a}$, where a is complex turns ratio, and

$a = |a|\angle r;\ \alpha = |\alpha|\angle\gamma; and\ r = -\gamma;\ y_{pq} = |y_{pq}|\angle\theta_{pq}$, then the power flow equations for the phase shifter are given by [refer to two winding transformer equations for analogy]

$$p_{pq} = Real\{E_t i_{tq}^*\} = |\alpha^2 E_p^2 g_{pq}| - |\alpha E_p E_q y_{pq}| cos(\gamma + \delta_p - \delta_q - \theta_{pq}) \quad (17b)$$
$$p_{qp} = Real\{E_q i_{qt}^*\} = |E_q^2 g_{pq}| - |\alpha E_p E_q y_{pq}| cos(\delta_q - \delta_p - \gamma - \theta_{pq}) \quad (17c)$$
$$q_{pq} = Imag\{E_t i_{tq}^*\} = -|\alpha^2 E_p^2 b_{pq}| - |\alpha E_p E_q y_{pq}| sin(\gamma + \delta_p - \delta_q - \theta_{pq}) \quad (17d)$$
$$q_{qp} = Imag\{E_q i_{qt}^*\} = -|E_q^2 b_{pq}| - |\alpha E_p E_q y_{pq}| sin(\delta_q - \delta_p - \gamma - \theta_{pq}) \quad (17e)$$

Note that equations 17b to 17e are similar to equations (15i) to (15l)

2.4.5.1 Partial Derivatives for Phase Shifter

We use the following basic differentiation identities in deriving the partial derivatives for power flow equation as function of the phase shifter angle γ

Function	Derivative
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$

$$\frac{\partial \mathbf{p_{pq}}}{\partial \boldsymbol{\gamma}} = |\boldsymbol{\alpha}\mathbf{E_p E_q y_{pq}}| \mathbf{sin}(\boldsymbol{\gamma} + \boldsymbol{\delta}_\mathbf{p} - \boldsymbol{\delta}_\mathbf{q} - \boldsymbol{\theta}_\mathbf{pq}) \quad \mathbf{(17f)}$$
$$\frac{\partial \mathbf{p_{qp}}}{\partial \boldsymbol{\gamma}} = -|\boldsymbol{\alpha}\mathbf{E_p E_q y_{pq}}| \mathbf{sin}(\boldsymbol{\delta}_\mathbf{q} - \boldsymbol{\delta}_\mathbf{p} - \boldsymbol{\gamma} - \boldsymbol{\theta}_\mathbf{pq}) \quad \mathbf{(17g)}$$
$$\frac{\partial \mathbf{q_{pq}}}{\partial \boldsymbol{\gamma}} = -|\boldsymbol{\alpha}\mathbf{E_p E_q y_{pq}}| \mathbf{cos}(\boldsymbol{\gamma} + \boldsymbol{\delta}_\mathbf{p} - \boldsymbol{\delta}_\mathbf{q} - \boldsymbol{\theta}_\mathbf{pq}) \quad \mathbf{(17h)}$$
$$\frac{\partial \mathbf{q_{qp}}}{\partial \boldsymbol{\gamma}} = |\boldsymbol{\alpha}\mathbf{E_p E_q y_{pq}}| \mathbf{cos}(\boldsymbol{\delta}_\mathbf{q} - \boldsymbol{\delta}_\mathbf{p} - \boldsymbol{\gamma} - \boldsymbol{\theta}_\mathbf{pq}) \quad \mathbf{(17i)}$$

2.4.6 Sensitivities of Bus Voltages to Bus Power Injections

The inverse of the load flow jacobian provides the required sensitivity of the dependent variables, x, with respect to the bus power injections. The bus power injections form the right hand side of the load flow jacobian equation [the mismatch vector].

PowerApps's jacobian matrix follows the formulation as per the reference [1]. The general structure between nodes 'k' and 'm' are denoted in the following equation.

$$\begin{bmatrix} \frac{\partial P_k}{\partial \delta_k} & \frac{\partial P_k}{\partial |E_k|}|E_k| & & \frac{\partial P_k}{\partial \delta_m} & \frac{\partial P_k}{\partial |E_m|}|E_m| \\ \frac{\partial Q_k}{\partial \delta_k} & \frac{\partial Q_k}{\partial |E_k|}|E_k| & & \frac{\partial Q_k}{\partial \delta_m} & \frac{\partial Q_k}{\partial |E_m|}|E_m| \\ & & & & \\ \frac{\partial P_m}{\partial \delta_k} & \frac{\partial P_m}{\partial |E_k|}|E_k| & & \frac{\partial P_m}{\partial \delta_m} & \frac{\partial P_m}{\partial |E_m|}|E_m| \\ \frac{\partial Q_m}{\partial \delta_k} & \frac{\partial Q_m}{\partial |E_k|}|E_k| & & \frac{\partial Q_m}{\partial \delta_m} & \frac{\partial Q_m}{\partial |E_m|}|E_m| \end{bmatrix} \begin{bmatrix} \Delta\delta_k \\ \frac{\Delta E_k}{E_k} \\ \\ \Delta\delta_m \\ \frac{\Delta E_m}{E_m} \end{bmatrix} = \begin{bmatrix} \Delta P_k \\ \Delta Q_k \\ \\ \Delta P_m \\ \Delta Q_m \end{bmatrix} \quad (18)$$

Notes:-

1. For slack bus the corresponding angle column does not exist as slack bus voltage is the reference bus. Correspondingly there is no P mismatch entry for slack bus.
2. For all PV bus where voltage control is possible, the corresponding column and corrections do not exist. Correspondingly there is Q mismatch entries $[\Delta Q_k]$and voltage magnitude correction entries $[\Delta E_k/E_k]$. This applies to Slack bus also as the slack bus voltage is a control variable.
3. The entries of the jacobian matrices are computed from equations 12 and 13.
4. The inverse of the jacobian matrix in equation (18) is the sensitivity matrix and provides the sensitivity of the bus voltage angle and magnitude with respect to the active and reactive power injection. Note we do not get the sensitivity with respect to slack generation ΔP_{sl} and reactive power generations, ΔQ_g as these are not modeled in the right hand side of the load flow jacobian equation [There is no mismatch power component for these]. Both these are dependent variables.
5. The jacobian matrix is structurally symmetric, but the values are not symmetric.
6. By providing 1.0 per unit power injection [or mismatch] at bus 'k' and keeping all other mismatch entries 0 in equation 18, and solving (18), we get the sensitivity of the entire system bus voltage magnitude and phase angle with respect to the injected 1.0 per unit power. Thus we can obtain sensitivity for 1.0 per unit active or reactive power. The resulting sensitivity solution is the corresponding column of the inverse of the load flow jacobian$\left[\frac{\partial g}{\partial x}\right]^{-1}$.

2.4.6.1 Sensitivities of Constant Bus Power Injections

The equation (18) provides the first part of equation (7), which is reproduced here.

$$\left[\frac{\partial g}{\partial x}\right]\Delta x + \left[\frac{\partial g}{\partial u}\right]\Delta u = 0 \quad (7)$$

The second part is the variation of the bus power injections with respect to the controllers which we have considered earlier.

2.4.6.2 Sensitivity of Bus Power Injection for Constant Impedance Type Compensation

In this case equation (14b) is applicable and is reproduced again from section 2.4.2

$$\frac{\partial Q_{sh}}{\partial B_{sh}} = -|E_{sh}|^2 \quad (14b)$$

This implies that the unity or '-1' value in equation (18a) is now replaced by $-|E_{sh}|^2$

2.4.6.3 Sensitivity of Bus Power Injection for Other controllers

2.4.6.3.1 Two Winding Transformers

The equations (15m) to (15p) of two winding transformers provides the entries relevant entries of equation (18a) , where there will be 4 entries for sending end P,Q and receiving end P,Q and contributes to the second part of the equation (7). The relevant portion are repeated here again

$$\frac{\partial p_{pq}}{\partial \tau} = 2\left|\tau E_p^2 g_{pq}\right| - \left|E_p E_q y_{pq}\right| cos\left(\delta_p - \delta_q - \theta_{pq}\right) \quad \text{(15m)}$$

$$\frac{\partial p_{qp}}{\partial \tau} = -\left|E_p E_q y_{pq}\right| cos\left(\delta_q - \delta_p - \theta_{pq}\right) \quad \text{(15n)}$$

$$\frac{\partial q_{pq}}{\partial \tau} = -2\left|\tau E_p^2 b_{pq}\right| - \left|E_p E_q y_{pq}\right| sin\left(\delta_p - \delta_q - \theta_{pq}\right) \quad \text{(15o)}$$

$$\frac{\partial q_{qp}}{\partial \tau} = -\left|E_p E_q y_{pq}\right| sin\left(\delta_q - \delta_p - \theta_{pq}\right) \quad \text{(15p)}$$

From equation (15m, 15n, 15o, 15p), it is seen that P,Q of “from” and “to” buses are affected. This information from equations (15m to 15p) is part of equation $\left[\frac{\partial g}{\partial u}\right] \Delta u$.

Similarly it should now be possible to relate bus power sensitivities of other controllers to $\left[\frac{\partial g}{\partial u}\right] \Delta u$ matrix. Most series elements, such as phase shifter, series compensated lines are similar to transformer in the sense these controllers affect P,Q flows like transformers.

2.4.6.3.2 Sensitivity of PQ bus power injection due to Generator Excitation

Generator excitation control $\left|E_g\right|$, influences the generator active power generation, reactive power generation and bus power injections of the connected buses. These are computed from equation 13.

However, we must note that generator active power generation is a specified quantity and do not change. Only the slack active power generation is dependent variable and entries related to this will be non-zero.

Thus in equation 13, $\frac{\partial P_k}{\partial |E_k|}|E_k|$ entries do not exist for PV buses [where k is generator bus]. But exist for slack bus. Equation (13) is reproduced in the following once again.

The partial derivatives when $k \neq m$, are given by

$$\frac{\partial P_k}{\partial \delta_m} = \frac{\partial Q_k}{\partial |E_m|}|E_m| = a_m f_k - b_m e_k \quad \text{(13a)}$$

$$-\frac{\partial Q_k}{\partial \delta_m} = \frac{\partial P_k}{\partial |E_m|}|E_m| = a_m e_k + b_m f_k \quad \text{(13b)}$$

The partial derivatives when $k = m$, are given by

$$\frac{\partial P_k}{\partial \delta_k} = -Q_k - B_{kk}|E_k|^2 \quad \text{(13c)}$$

$$\frac{\partial Q_k}{\partial \delta_k} = P_k - G_{kk}|E_k|^2 \quad \text{(13d)}$$

$$\frac{\partial P_k}{\partial |E_k|}|E_k| = P_k + G_{kk}|E_k|^2 \quad \text{(13e)}$$

$$\frac{\partial Q_k}{\partial |E_k|}|E_k| = Q_k - B_{kk}|E_k|^2 \quad \text{(13f)}$$

Since Generator Excitation corresponds to Slack bus and PV bus in PowerApps load flow, the elements of partial derivatives with respect to slack bus voltage angle and with respect to generator excitation will not exist in regular load flow. Consequently we need to temporarily convert the slack bus and the PV buses to PQ buses and formulate the jacobian. This jacobian will now contain all related sensitivities and required partial derivatives must now be extracted from this jacobian. For identifying and differentiating this jacobian from load flow jacobian, wherein all buses are considered as PQ buses we can call this as PQjacobian instead of load flow jacobian. Note the PQjacobian will also have information about generator reactive power output partial derivative with respect to generator excitation controls which forms part of H matrix for security constraints.

2.4.7 Calculation of the Slack Bus Generation Sensitivity

Note that **Slack** Generation ΔP_k does not exist in equation (18) in the load flow jacobian. Consequently we need to compute this separately as follows

We repeat the equations 12 and 13 from section 2.4.1.

$Y_{km} = (G_{km} + B_{km});\ E_m = (e_m + f_m);\ \ and\ I_m = (a_m + b_m);$ (12a, b and c)

The partial derivatives when$k \neq m$, are given by

$$\frac{\partial P_k}{\partial \delta_m} = \frac{\partial Q_k}{\partial |E_m|}|E_m| = a_m f_k - b_m e_k \qquad (13a)$$

$$-\frac{\partial Q_k}{\partial \delta_m} = \frac{\partial P_k}{\partial |E_m|}|E_m| = a_m e_k + b_m f_k \qquad (13b)$$

The partial derivatives when$k = m$, are given by

$$\frac{\partial P_k}{\partial \delta_k} = -Q_k - B_{kk}|E_k|^2 \qquad (13c)$$

$$\frac{\partial Q_k}{\partial \delta_k} = P_k - G_{kk}|E_k|^2 \qquad (13d)$$

$$\frac{\partial P_k}{\partial |E_k|}|E_k| = P_k + G_{kk}|E_k|^2 \qquad (13e)$$

$$\frac{\partial Q_k}{\partial |E_k|}|E_k| = Q_k - B_{kk}|E_k|^2 \qquad (13f)$$

Now assume that the bus ‘k’ in the equations refers to slack bus and bus ‘m’ refers to non-slack bus. Further assume bus ‘m’ is not a PV bus [i.e. voltage controlled bus, meaning slack bus is not directly connected to PV bus].

Equations (13a) and (13b) give the variation of the slack generation [in this case ‘Pk’] with respect to the bus voltage angle and bus voltage magnitude of bus ‘m’.

Equation (13c) is not valid for slack bus, whose angle ‘δ_k’ is the reference angle and is typically 0 in value.

Typically slack bus voltage magnitude is a control variable and belongs to vector ‘u’ and equation (13e) provides the necessary sensitivity of the slack generation with respect to its own excitation control ‘Ek’.

Note that generator transformer taps are not part of control variables. The effect of generator taps can be handled through ‘Ek’. Thus the slack bus generation via its own generator transformer is ignored. [Do not represent generator transformer tap as control variables in the input data]

The slack bus generation is expressed in terms of the control [u] and dependent [x] variables as follows

$$\Delta P_{sl} = \frac{\partial P_{sl}}{\partial u}\Delta u + \frac{\partial P_{sl}}{\partial x}\Delta x \qquad (19a)$$

$$\Delta P_{sl} = \frac{\partial P_{sl}}{\partial u}\Delta u + \frac{\partial P_{sl}}{\partial x}\left[S_x\right]\Delta u \qquad (19b)$$

$$\Delta P_{sl} = \left[\frac{\partial P_{sl}}{\partial u} + \frac{\partial P_{sl}}{\partial x}\left[S_x\right]\right]\Delta u \qquad (19c)$$

Equation (19c) is the required cost function, when the objective of the optimization is minimization of the slack generation or minimization of the transmission loss.

2.4.8 Calculation of Sensitivities of the Reactive power Generation

Similar to slack generation, the reactive power generation ΔQ_k does not exist in equation (18). We therefore have to use the equations 12 and 13 again to compute the sensitivities in the following format similar to slack generation

$$\Delta Q_g = \left[\frac{\partial Q_g}{\partial u} + \frac{\partial Q_g}{\partial x}\left[S_x\right]\right]\Delta u \qquad (20)$$

Equation (20) is part of the security limits in the LP Tableau formulation.

Notes:

1. Usually the only control variable that influences a specific generator reactive power generation is the corresponding generator excitation $|E_g|$ which belongs to the vector u. [Equation (13f) is applicable in this case]. No other controller is likely influence reactive power generation, unless they are directly connected to the Generator bus, which appears unlikely. Generator transformer tap is usually not considered as a control variable]
2. The second term related to dependent variable influence on changes in generator reactive power is given by equations (13a), (13b), and (13d). [Equation (13d) is not applicable for slack bus, as slack voltage angle does not change]

2.4.9 Line/Transformer Flows Security constraints

Line/Transformer flow constraints are modeled by equation 11(b)

$$\Delta h = \left[\left[\frac{\partial h}{\partial x}\right]\left[S_x\right] + \left[\frac{\partial h}{\partial u}\right]\right]\Delta u \qquad (11b)$$

The elements of the first matrix in (11b) are related to dependent variables [bus voltage magnitudes, phase angles of PQ buses, fixed tap positions or variables on the left hand side of equation (18)].

The elements of the second matrix in (11b) are related to control variables that directly influences the power flows in concerned line/transformer/phase shifter/series compensator etc.

Constraints on line/transformer flows are implemented based on equations given for transformers. The tap $\tau = 1$ for lines as tap is not applicable in case of lines. The elements of (11b) are computed from the following equations which are reproduced here.

$$p_{pq} = Real\{E_t i_{tq}^*\} = \left|\tau^2 E_p^2 y_{pq}\right|\cos(\theta_{pq}) - \left|\tau E_p E_q y_{pq}\right|cos(\delta_p - \delta_q - \theta_{pq}) \qquad (15i)$$

$$p_{qp} = Real\{E_q i_{qt}^*\} = \left|E_q^2 y_{pq}\right|cos(\theta_{pq}) - \left|\tau E_p E_q y_{pq}\right|cos(\delta_q - \delta_p - \theta_{pq}) \qquad (15j)$$

$$q_{pq} = Imag\{E_t i_{tq}^*\} = -\left|\tau^2 E_p^2 y_{pq}\right|\sin(\theta_{pq}) - \left|\tau E_p E_q y_{pq}\right|sin(\delta_p - \delta_q - \theta_{pq}) \qquad (15k)$$

$$q_{qp} = Imag\{E_q i_{qt}^*\} = -\left|E_q^2 y_{pq}\right|\sin(\theta_{pq}) - \left|\tau E_p E_q y_{pq}\right|sin(\delta_q - \delta_p - \theta_{pq}) \qquad (15l)$$

The partial derivative equations of the flows with respect to the variable τ is given by the following

$$\frac{\partial p_{pq}}{\partial \tau} = 2\left|\tau v_p^2 g_{pq}\right| - \left|v_p v_q y_{pq}\right| cos\left(\delta_p - \delta_q - \theta_{pq}\right) \qquad (15m)$$

$$\frac{\partial p_{qp}}{\partial \tau} = -\left|v_p v_q y_{pq}\right| cos\left(\delta_q - \delta_p - \theta_{pq}\right) \qquad (15n)$$

$$\frac{\partial q_{pq}}{\partial \tau} = -2\left|\tau v_p^2 b_{pq}\right| - \left|v_p v_q y_{pq}\right| sin\left(\delta_p - \delta_q - \theta_{pq}\right) \qquad (15o)$$

$$\frac{\partial q_{qp}}{\partial \tau} = -\left|v_p v_q y_{pq}\right| sin\left(\delta_q - \delta_p - \theta_{pq}\right) \qquad (15p)$$

2.4.9.1 Notes on Transmission Line/Transformer/Phase Shifter/Series Compensation Load Constraints

For normal lines, $\tau = 1$, [not a control variable]. For transformer τ exists as control variable portion [u]. For phase shifter additional variables will exist [phase shift angle]. For TCSC series compensation will exist as control variable.

Apart from the control variable, if the circuit in question is directly connected to a generator bus, the generator bus voltage as control variable will exist in the formulation of $\left[\frac{\partial h}{\partial u}\right]$.

In equations (15i) to (15p), (15m) to (15p) type of equations with respect to controllers will exist only if either bus 'p' or bus 'q' is a generator bus or y_{pq} refers to variable series compensation or $\boldsymbol{\theta_{pq}}$, has part of phase shifter angle. In the absence of these there will be no entries related to $\left[\frac{\partial h}{\partial u}\right]$ matrix, of equation (11b).

The entries of $\left[\frac{\partial h}{\partial x}\right]$ will exist in (11b) related to variables belonging to dependent variables [x].

3 Simplex Tableau Formulation

We repeat some of the equations and show the structure of the final Simplex Tableau for the OPF problem

Minimize $f(x,u)$ [cost function] (1)

Subject to constraints

$g(x,u) = 0$ [Load flow constraints] (2)

$umin \leq u \leq umax$ [Controller minimum and maximum limits] (3)
$xmin \leq x \leq xmax$ [Dependent variable minimum and maximum limits] (4)
$hmin \leq h \leq hmax$ [Security Constraints] (5)

But in the above equations we have

$$\Delta x = -\left[\frac{\partial g}{\partial x}\right]^{-1}\left[\frac{\partial g}{\partial u}\right]\Delta u \qquad (8a)$$

$$\Delta x = [S_x]\Delta u \qquad (8b)$$

and for loss minimization objective, we have

$$f(x,u) = \Delta P_{sl} = \left[\left[\frac{\partial P_{sl}}{\partial x}\right]^T [S_x] + \left[\frac{\partial P_{sl}}{\partial u}\right]^T\right]\Delta u = [C^T]\Delta u \qquad (9c)$$

and for security constraints we have

$$\Delta h = \left[\left[\frac{\partial h}{\partial x}\right][S_x] + \left[\frac{\partial h}{\partial u}\right]\right]\Delta u = [H]\Delta u \tag{11b}$$

With the above, we may state the minimization problem as follows

Minimize $f(x,u) = \Delta P_{sl} = \left[\left[\frac{\partial P_{sl}}{\partial x}\right]^T [S_x] + \left[\frac{\partial P_{sl}}{\partial u}\right]^T\right]\Delta u = [C^T]\Delta u$ (9c)

subject to

$$\begin{aligned} [S_x]\Delta u &\geq \Delta x_{min} \\ -[S_x]\Delta u &\geq -\Delta x_{max} \\ \Delta u &\geq \Delta u_{min} \\ -\Delta u &\geq -\Delta u_{max} \\ [H]\Delta u &\geq \Delta h_{min} \\ -[H]\Delta u &\geq -\Delta h_{max} \end{aligned}$$

However in actual implementation we Maximize the negative of (9c) and change the limits to ≤ instead of ≥, consequently our problem becomes

Maximize

$-f(x,u) = -\Delta P_{sl} = -\left[\left[\frac{\partial P_{sl}}{\partial x}\right]^T [S_x] + \left[\frac{\partial P_{sl}}{\partial u}\right]^T\right]\Delta u = -[C^T]\Delta u$ [22a]

subject to

-$[S_x]\Delta u \leq -\Delta x_{min}$ [22b]
$[S_x]\Delta u \leq \Delta x_{max}$ [22c]
$-\Delta u \leq -\Delta u_{min}$ [22d]
$\Delta u \leq \Delta u_{max}$ [22e]
$-[H]\Delta u \leq -\Delta h_{min}$ [22f]
$[H]\Delta u \leq \Delta h_{max}$ [22h]

Equation [22] may be put in matrix notation or simplex tableau as follows [23 Tableau]

$-[S_x]$	$-\Delta x_{min}$
$[S_x]$	Δx_{max}
$-[H]$	$-\Delta h_{min}$
$[H]$	Δh_{max}
$-I$	$-\Delta u_{min}$
I	Δu_{max}
$-[C^T]$	0

[22i]

The 0 in the bottom right cell indicates that the initial value of the objective function is 0.

Student must be able to compute the individual matrices of the tableau 23 and formulate the tableau of [23] using class CPFCMat

3.1 Size of the LP Tableau

Equation [22i] indicates the organization of the LP Tableau. What is its size?

A. The number of columns in the tableau is = number of control variables + 1. If Nu is the number of control variables considered for the OPF problem, then, the number of columns in the LP tableau is Nu+1. The array of CPFCControlvar class in CPFCCase will give the number of control variables considered for the OPF studies.
B. The number of rows in Sx = number of dependent variables having constraints. Assuming there are constraints only on bus voltage magnitude, this number is equal to the number of PQ buses [of bus type 3]. Thus if Ngen is the number of generators, Nbus is the number of buses, the number of PQ buses are (Nbus-Ngen)
C. The number of rows of [H] matrix is equal to the number of security constraints specified. It becomes expensive to consider all possible line flow constraints. As a minimum we may consider only generator reactive power limits only, in which case the we will have Ngen number of rows in [H] matrix. If line flow constraints are also considered [as may be needed for ABT solutions], the number of rows in [H] will increase by the number of line flow constraints considered. Student may therefore provide an input option for which line flows needs to be considered.
D. The identity matrix in the tableau is diagonal and has same number of rows and columns as the number of controllers Nu.
E. The total number of rows in the LP tableau is twice the number of constraints as upper and lower limits are represented separately. In addition there is one more bottom row which stores the cost coefficient of the objective function.

We may therefore summarize the size of LP Tableau as follows

Number of columns = Number of Controllers considered + 1
Number of rows = 2*(Number of PQ buses + Number of security constraints + Number of controllers) + 1

4 References

1. William F.Tinney, Clifford E.Hart, “Power flow solution by Newton’s method”, IEEE transactions on Power Apparatus and Systems, Vol. PAS-86, No. 11, November 1967.
2. Hermann W. Dommel, William F.Tinney, “Optimal Power Flow Solutions”, IEEE transactions on Power Apparatus and Systems, Vol. PAS-87, No. 10, October 1968.